Name:________________________ Maths Class:____________________

SYDNEY TECHNICAL HIGH SCHOOL

TRIAL HIGHER SCHOOL CERTIFICATE

2004

MATHEMATICS

Time allowed: ***3 hours plus 5 mins reading time***

Instructions:

- Write your name and class at the top of this page, and at the top of each answer sheet.
- At the end of the examination this examination paper must be attached to the front of your answers.
- All questions are of equal value and may be attempted.
- All necessary working must be shown. Marks will be deducted for careless or badly arranged work.
- Marks indicated are a guide only and may be varied if necessary.
- Non programmable calculators may be used.

(For markers use only)

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Total

Question 1 **Marks**

a) Factorise fully $16x^2 - 81$ **1**

b) Convert $\frac{4\pi}{5}$ radians to degrees **1**

c) Given $f(x) = 1 - x^3$, find x when $f(x) = 65$ **1**

d) Find the values of a and b if $\frac{1}{2\sqrt{3}-1} = a + b\sqrt{3}$ **2**

e) Find the exact value of $\tan\ 300°$ **2**

f) Evaluate $\lim_{x \to -2} \frac{3x^2 + 7x + 2}{x + 2}$ **2**

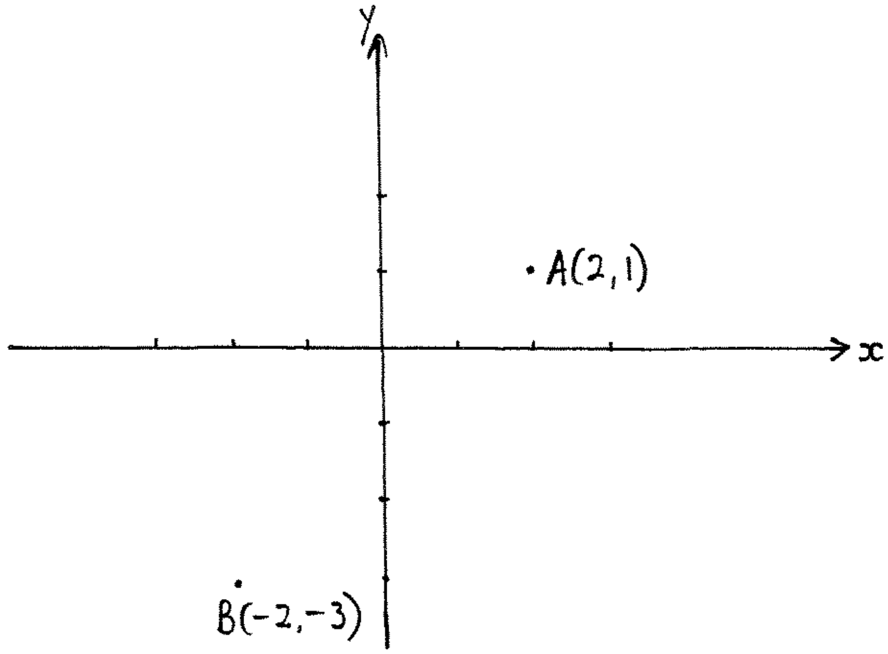
g) Solve and graph the solution on a number line: $|6x - 9| > 21$ **3**

Question 2 (Begin a new page)

a) The roots of the quadratic equation $3x^2 + 4x + 2 = 0$ are α *and* β.

Find the value of $2\alpha\beta^2 + 2\alpha^2\beta$ **3**

b) **Marks**



(i) Show that the distance between A and B is $4\sqrt{2}$ units. **2**

(ii) Find the mid-point C, of AB **1**

(iii) Show that the gradient of AB is 1 **1**

(iv) Show that the line through C perpendicular to AB has equation $x + y + 1 = 0$ **2**

(v) Show that this line passes through D (-3, 2) **1**

(vi) Find the area of ΔABD **2**

Question 3 (Begin a new page)

a) Differentiate the following with respect to x:

(i) $x^2 + \sqrt{x}$ **1**

(ii) $x^2 \tan x$ **2**

(iii) $\sin(e^x)$ **2**

b) Find (i) $\int \frac{x^2}{x^3 - 2} dx$ **2**

(ii) $\int e^{3x} dx$ **1**

Marks

c) Evaluate

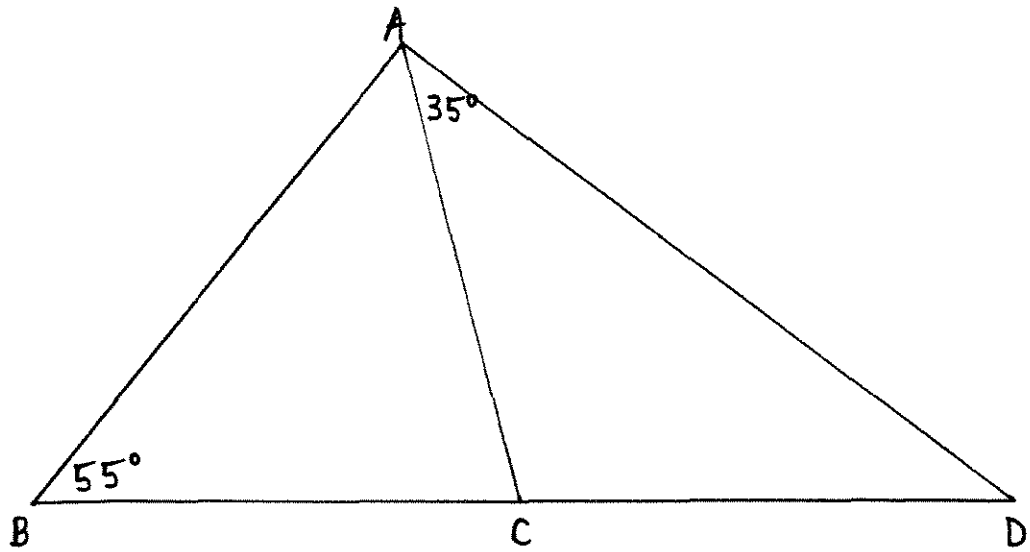
(i) $\int_0^1 (2x+1)^5 \, dx$ **2**

(ii) $\int_0^{\frac{\pi}{4}} \sin 2x dx$ **2**

Question 4 (Begin a new page)

a) Find the values of k for which $x^2 + kx + 16$ is positive definite **2**

b)



2

Given that $AC = DC, \angle ABC = 55°$ and that $\angle DAC = 35°$, show that triangle ABC is isosceles.

c) Consider the curve whose equation is $y = x^3 - 12x + 5$.

(i) Find the coordinates of the stationary points. **3**

(ii) Determine the nature of the stationary points. **1**

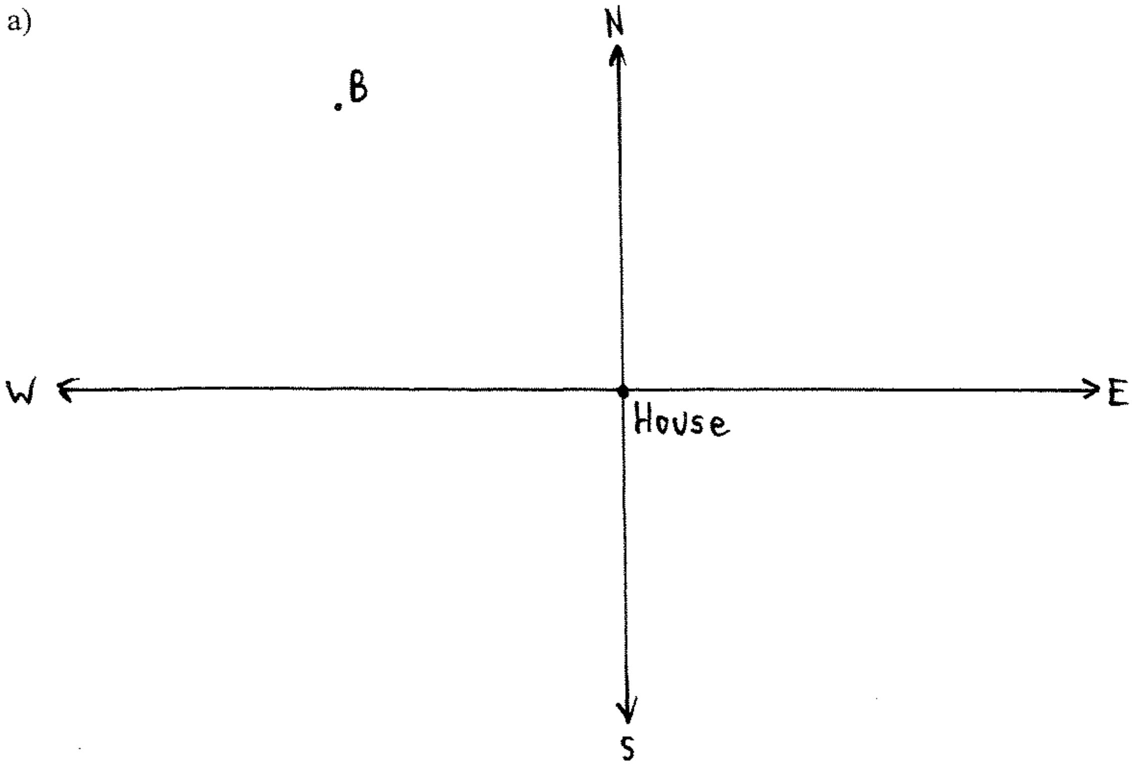
(iii) Find the point of inflexion. **2**

(iv) Sketch the curve over the domain $-3 \le x \le 3$.(x intercept not required) **1**

(v) Find the minimum value of the function over this domain. **1**

Question 5 (Being a new page) **Marks**

a) 3



Samantha walks from her house for 6km, on a bearing of $310°$ to point B. She then walks on a bearing of $215°$ until she is due west of the house. How far is she now from her house? (correct to one decimal place).

b) The number, N, of people with flu is increasing over time t. Also, the rate at which people are catching flu is increasing.

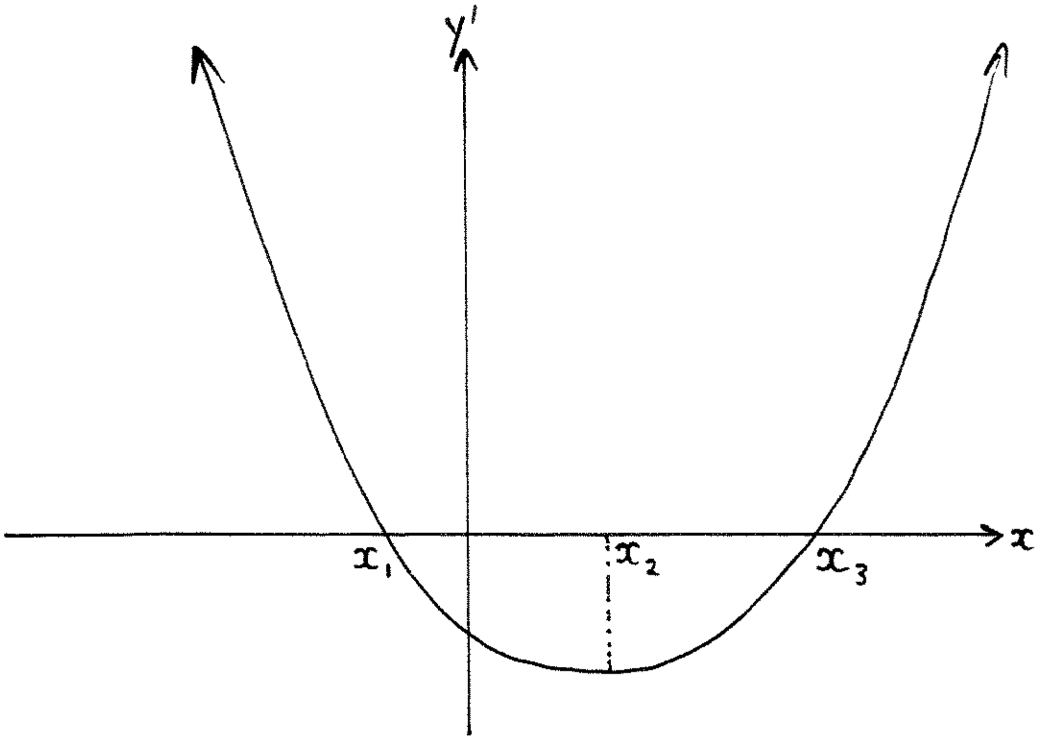
(i) State the sign (+ or -) of $\frac{dN}{dt}$ *and* $\frac{d^2N}{dt^2}$ **1**

(ii) Sketch a possible graph of N = $f(t)$ which illustrates this information. **1**

c) Given that $\tan A = P$, and $180° < A < 270^0$, find an expression for cos A in terms of P. **2**

d) If $\int_0^a (4-2x)dx = 4$, find the value of a. **2**

e) Find the x value of the point on the parabola $y = x^2 + x - 1$ where the tangent is parallel to the line $y = 9x - 5$. **2** f)



1

The sketch above shows the derivative function for a certain curve. Copy this diagram into your answer booklet and on it, sketch a curve that could be the original function.

Question 6 (Begin a new page)

a) Consider the parabola with equation $x^2 = 8(y-2)$.

(i) Find the coordinates of the vertex **1**

(ii) Find the coordinates of the focus **1**

(iii) Find the exact volume of the solid formed (a paraboloid) if the portion of the parabola from $y = 2$ to $y=4$ is rotated about the y axis. **2**

b) To what sum will \$ 4500 amount if invested at 10% p.a. for 6 years if the interest is compounded quarterly? **2** c)

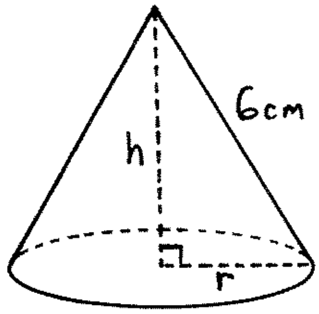
t	0	5	10	15	20
T	83	74	63	50	41

2

The table above shows the temperature $T°$ of an object cooling down over t minutes.

If T = f(t),use all the values in this table, to approximate $\int_0^{20} f(t)dt$ with the Trapezoidal Rule.

d) The slant edge of a right circular cone of height 'h' and base radius 'r' cm, is 6cm.



(i) Write down an equation linking r and h. **1**

(ii) Given that the formula for the volume of a cone is $V = \frac{1}{3}\pi r^2 h$, **1**

use part (i) or otherwise to show $V = 12\pi h - \frac{1}{3}\pi h^3$.

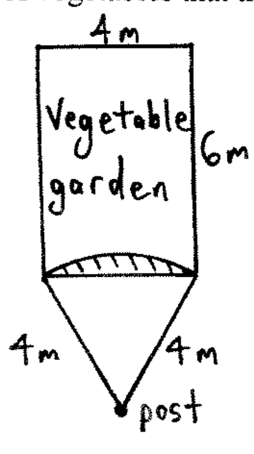
(iii) Hence find the height of the cone which gives a maximum volume. **2**

Question 7 (Begin a new page)

a) Solve $25^k (5^3)^4 = 1$ **2**

b) A goat is tethered to a 4 metre long rope. The other end of the rope is tied to a post fixed at a point 4 metres from each of two corners of a 6 metre by 4 metre rectangular vegetable garden. This information is illustrated in the **diagram below.** **3**

Calculate the exact area of vegetables that the goat can eat.



Marks

c) Evaluate $\sum_{n=3}^{n=12} (2 \times 3^n)$ leaving your answer in index form. **2**

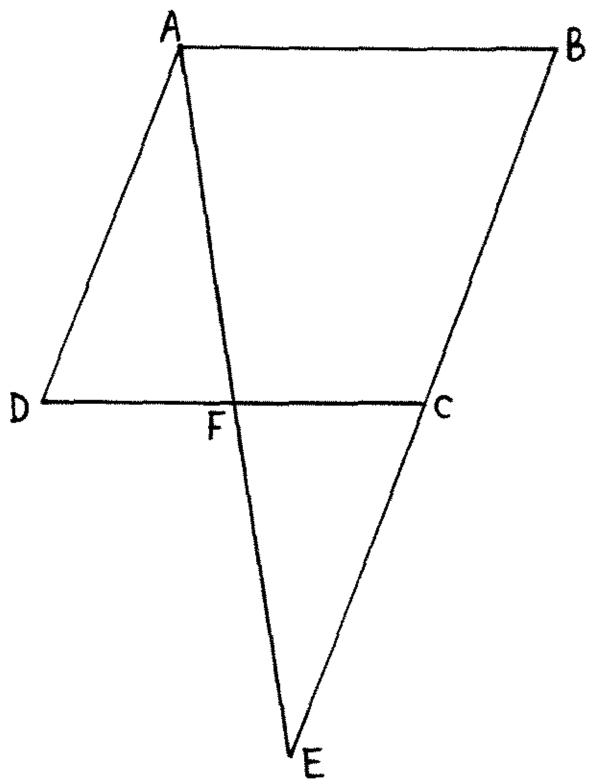
d) (i) Draw a neat sketch of the graph of: $f(x) = -2\sin x \quad for \quad -\pi \le x \le \pi$ **2**

(ii) Show that it is an odd function. **1**

(iii) Hence or otherwise calculate the area bounded by the above curve, the x – axis and between $x = -\pi \quad and \quad x = \pi$. **2**

Question 8 (Begin a new page)

a)



The figure above shows a rhombus ABCD with BC produced to E so that BC=CE.

Copy this diagram onto your answer page

(i) Prove that triangles ADF and EBA are similar. **2**

(ii) Prove that F is the midpoint of DC. **2**

Marks

b) A chemical substance being made in a laboratory decomposes and the amount M in kilograms present at any time t hours is given by $M = M_o e^{-kt}$.

If $\frac{3}{4}$ of the mass of this substance will disintegrate in 4 hours, find:

(i) the value of k. correct to two decimal places. **2**

(ii) the value of M_0 given 4kg of the substance remains after 90 minutes, correct to two decimal places. **2**

c) A particle moving in a straight line with a constant acceleration of $6\,m/s^2$, is initially at $x = 2$ with a velocity of *2 m/s*.

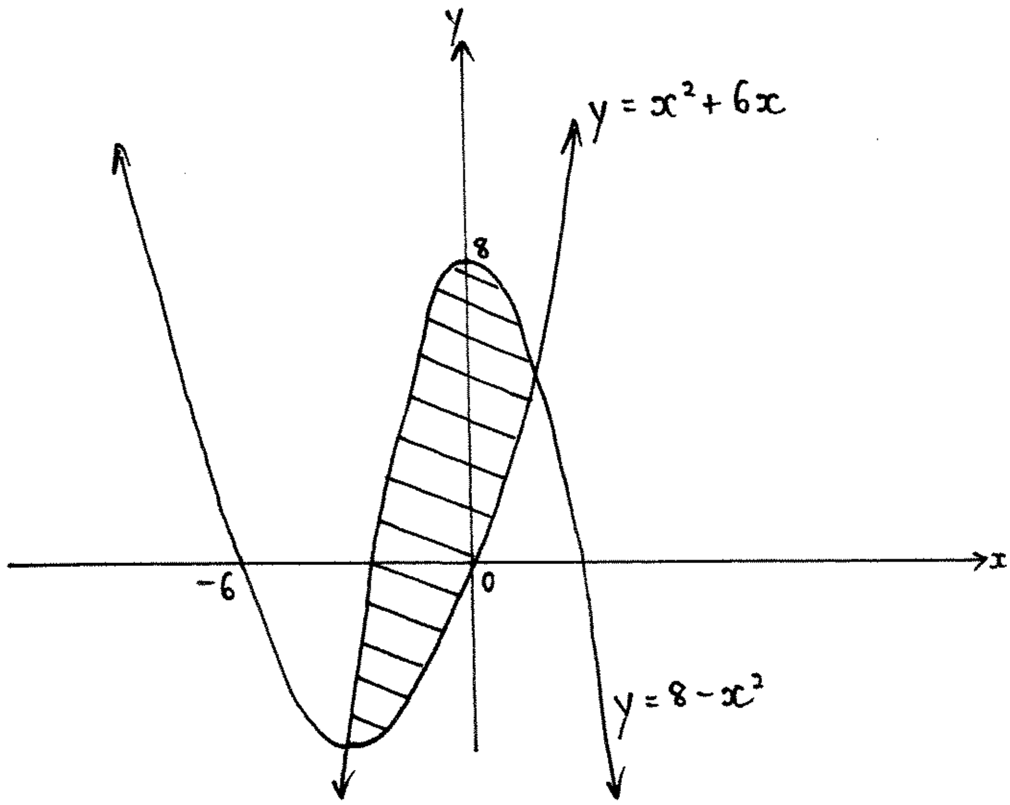
(i) Calculate its velocity and displacement in terms of t. **2**

(ii) Draw a velocity time graph for the first four seconds. **1**

(iii) Hence or otherwise find the total distance travelled during the first 4 seconds. **1**

Question 9 (Being a new page)

a)



Calculate the area of the shaded region above. **3**

Marks

b) The second term of a geometric series is 27 and the fifth term is 64.

(i) Find the first term and the common ratio. **2**

(ii) Find the sum of the first five terms of this series. **2**

c) Show that if $y = \ln\left(\frac{2x+1}{3x-1}\right)$, then $\frac{dy}{dx} = -\frac{5}{(2x+1)(3x-1)}$ **2**

d) Solve $\cos^2 2x = \frac{1}{4}$ for $0 \le x \le 360°$ **3**

Question 10 (Begin a new page)

a) A square metal plate, with an original side length of 20cm, is being heated so that the length 'L' of each side of the plate at any time 't' is
L = 4 t + 20.

(i) Find an expression for the area of the plate at time 't' seconds. **1**

(ii) After what time has the area of the plate reached $784cm^2$? **2**

(iii) Find the rate of increase of the area when t = 1 second. **2**

b) Bill borrows $100000 at 6% p.a. monthly reducible, to be repaid monthly over 10 years.

(i) Given he pays $P per month, and the amount owing after n months is $\$ A_n$, show that after 2 months, the amount owing is

$A_2 = 100000(1.005)^2 - P(1 + 1.005)$ **2**

(ii) Hence show that the amount owing after n months is:

$A_n = 100000(1.005)^n - 200P(1.005^n - 1)$ **3**

(iii) Calculate to the nearearst cent,the monthly repayment required to repay the loan in10 years. **2**

Teacher's Name: ______________ **Student's Name/Nº:** ______________

Solutions to 2004 S.T.H.S 2U Trial HSC

Question 1

(a) $16x^2 - 81$

$(4x-9)(4x+9)$ (1)

(b) $\frac{4\pi}{5} \times \frac{180}{\pi}$

$= 144^\circ$ (1)

(c) $f(x) = 1 - x^3$

$f(x) = 65 = 1 - x^3$

$x^3 = -64$

$x = -4$ (1)

(d) $\frac{1}{2\sqrt{3}-1} \times \frac{2\sqrt{3}+1}{2\sqrt{3}+1}$

$= \frac{2\sqrt{3}+1}{12-1}$

$= \frac{2\sqrt{3}+1}{11}$

$= \frac{1}{11} + \frac{2}{11}\sqrt{3}$

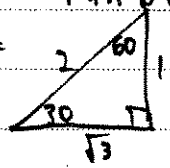
$a = \frac{1}{11}$, $b = \frac{2}{11}$

(1) (1)

(e) $\tan 300$

(1) $= -\tan 60$

$=$



(1) $= -\sqrt{3}$

(f) $\lim_{x \to -2} \frac{3x^2+7x+2}{x+2}$

$= \lim_{x \to 2} \frac{(3x+1)\cancel{(x+2)}}{\cancel{x+2}}$ (1)

$= -5$ (1)

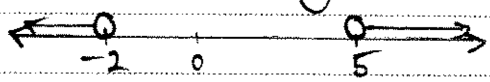
(g) $|6x-9| > 21$

$6x - 9 > 21$, $6x - 9 < -21$

$6x > 30$, $6x < -12$

$x > 5$ (1) $x < -2$ (1)

(1)



Question 2.

(a) $3x^2 + 4x + 2 = 0$

$2\alpha\beta^2 + 2\alpha^2\beta$

$= 2\alpha\beta(\beta + \alpha)$

$= 2 \times \frac{c}{a} \times \frac{-b}{a}$ (1)

$= 2 \times \frac{2}{3} \times \frac{-4}{3}$ (1)

$= \frac{-16}{9}$ (1)

(b) (i) $AB = \sqrt{(2--2)^2 + (1--3)^2}$ (1)

$= \sqrt{4^2 + 4^2}$

$= \sqrt{32}$

$= 4\sqrt{2}$ units (1)

(ii) $C = \left(\frac{-2+2}{2}, \frac{-3+1}{2}\right)$

$= (0, -1)$ (1)

(iii) $m_{AB} = \frac{1--3}{2--2}$

$= \frac{4}{4}$

$= 1$ (1)

(iv) $\therefore \perp m_{AB} = -1$ $\rightarrow C(0,-1)$

$y - y_1 = m(x - x_1)$ (1)

$y - -1 = -1(x - 0)$

$x + y + 1 = 0$ as required (1)

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(v) $D(-3,2)$ lies on $x+y+1=0$ if it satisfies equation.

$-3+2+1=0$ ✓ $\therefore$ Yes

(1)

(vi) $A=\frac{1}{2}bh$

$=\frac{1}{2}\times AB\times CD$

$=\frac{1}{2}\times 4\sqrt{2}\times CD$

$CD=$ d. bet. $(0,-1)$ and $(-3,$

$=\sqrt{(-3-0)^2+(2--1)^2}$

$=\sqrt{9+9}$

$=3\sqrt{2}$ (1)

$A=\frac{1}{2}\times 4\sqrt{2}\times 3\sqrt{2}$

$=12$ units^2 (1)

Question 3. (a) (i) $x^2+\sqrt{x}$

$\frac{d}{dx}(x^2+x^{\frac{1}{2}})$

$=2x+\frac{1}{2}x^{-\frac{1}{2}}$ (1)

(ii) $x^2\tan x$ (1)

$\frac{d}{dx}=x^2\sec^2x+\tan x\times 2x$

$=x^2\sec^2x+2x\tan x$ (1)

(iii) $\sin(e^x)$

$\frac{d}{dx}=\cos e^x\times e^x$ (1)

$=e^x\cos(e^x)$ (1)

(b) (i) $\int\frac{x^2}{x^3-2}\,dx$

$=\frac{1}{3}\int\frac{3x^2}{x^3-2}\,dx$ (1)

$=\frac{1}{3}\ln(x^3-2)+c$ (1)

(ii) $\int e^{3x}\,dx$

$=\frac{1}{3}\int 3e^{3x}\,dx$

$=\frac{1}{3}e^{3x}+c$ (1)

(c)(i) $\int_0^1(2x+1)^5\,dx$

$\left[\frac{(2x+1)^6}{6\times 2}\right]_0^1$ (1)

$=\frac{3^6-1^6}{12}$

$=60\frac{2}{3}$ (1)

c(ii) $\int_0^{\frac{\pi}{4}}\sin 2x\,dx$

$\left[-\frac{1}{2}\cos 2x\right]_0^{\frac{\pi}{4}}$ (1)

$-\frac{1}{2}\left(\cos\frac{\pi}{2}-\cos 0\right)$

$=-\frac{1}{2}(0-1)$

$=\frac{1}{2}$ (1)

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Question 4

ⓐ $x^2 + kx + 16$

$\Delta < 0$

$k^2 - 4 \times 1 \times 16 < 0$ ①

$k^2 - 64 < 0$

$(k-8)(k+8) < 0$

$\underline{-8 < k < 8}$ ①

ⓑ Since ΔACD is isosceles,

$\angle ACB = 2 \times 35$

$= 70°$ (exterior angle of a triangle)

$\therefore \angle BAC = 180 - 55 - 70$ ①

$= 55°$ (Angle sum of a triangle)

$\Rightarrow \Delta ABC$ is isosceles as it contai[ns] 2 equal angles. ①

ⓒ (i) $y = x^3 - 12x + 5$

Stat. pts. where $y' = 0$

$y' = 3x^2 - 12 = 0$ ①

$x^2 - 4 = 0$

$x = \pm 2$

$\therefore$ (2, −11) ① (−2, 21) ① are the stationary points

(ii) $y'' = 6x$

When $x = 2$, $y'' > 0$

$\therefore$ (2, −11) is a minimum turning pt. ①

When $x = -2$, $y'' < 0$

$\therefore$ (−2, 21) is a maximum turning pt.

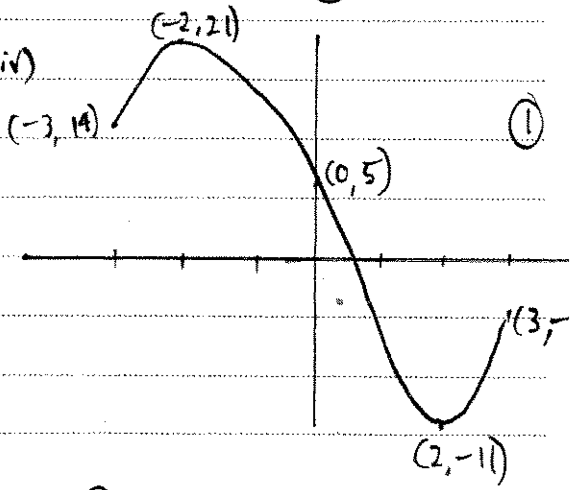
(iii) Pt. of inflexion occurs when $y'' = 0$

$6x = 0$ ①

when $x = 0$ must be a pt. of inflexion (non horizontal) as $y' \neq 0$ at this point

$\therefore$ (0, 5) is the inflexion ①

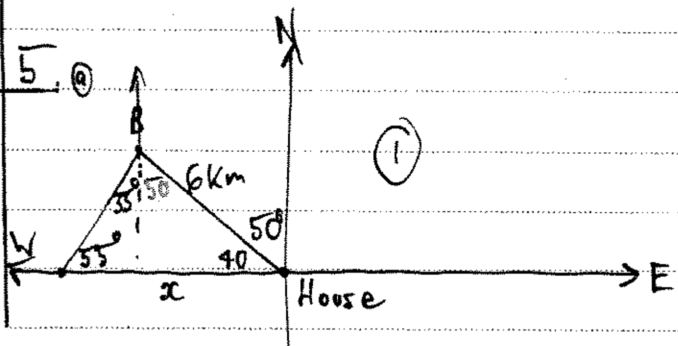
(iv)



①

(v) −11 is the minimum value ①

Question 5. ⓐ



①

$\dfrac{x}{\sin 85} = \dfrac{6}{\sin 55}$ ①

$x = 7.3\,km$ ①

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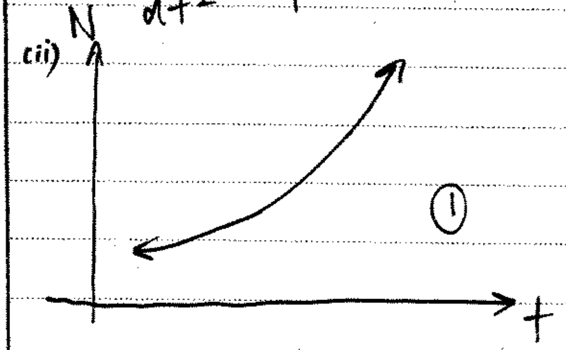
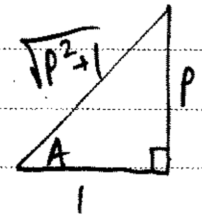
(b) (i) $\frac{dN}{dt}$ $+$, $\frac{d^2N}{dt^2}$ $+$ (1)

(ii)



(1)

(c)



(1)

(1)

$\cos A = \frac{-1}{\sqrt{p^2+1}}$ as A is in 3rd quadra.

(d) $\int_0^a 4 - 2x \, dx = 4$

$\left[4x - x^2\right]_0^a = 4$

$4a - a^2 - 0 = 4$ (1)

$a^2 - 4a + 4 = 0$

$(a-2)^2 = 0$

$a = 2$ (1)

(e) $y = x^2 + x + 1$

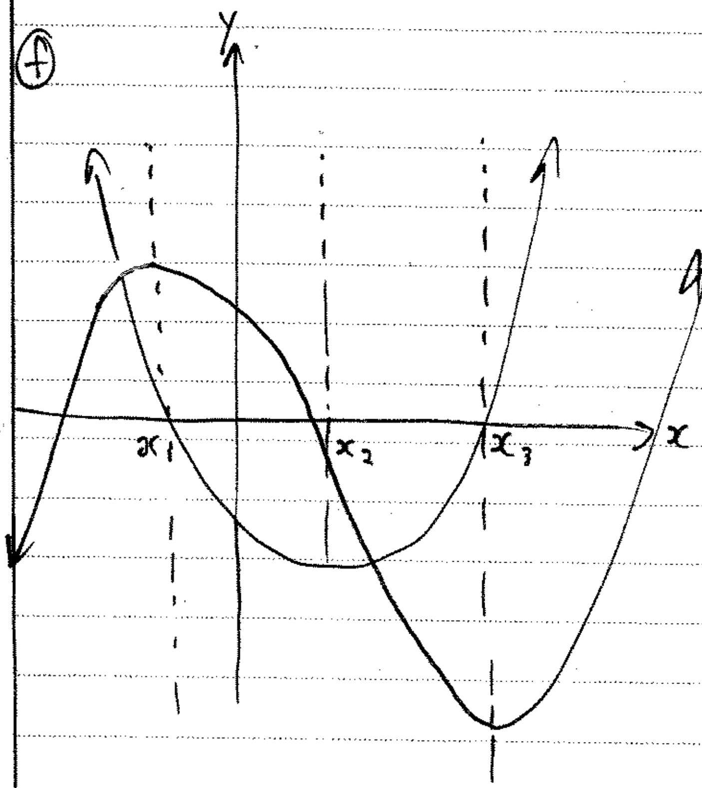
$\frac{dy}{dx} = 2x + 1$ parallel to 9x

$\Rightarrow 2x + 1 = 9$ (1)

$2x = 8$

$x = 4$ (1)

(f)



Stationary pts. must line up with x_1 and x_3.

(1)

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Question 6 (a) $x^2 = 8(y-2)$

(i) Vertex $(0,2)$ ①

(ii) Focus $(0,4)$ ①

(iii) $V = \pi\int x^2 \, dy$

$= \pi\int_2^4 8(y-2)\, dy$

$= 8\pi\left[\frac{y^2}{2} - 2y\right]_2^4$ ①

$= 8\pi[8 - 8 - (2-4)]$

$= 16\pi$ units^3 ①

(b) $A = P(1+\frac{r}{100})^n$

$r = 10 \div 4 = 2.5$

$n = 6 \times 4 = 24$

$\therefore A = 4500(1+\frac{2.5}{100})^{24}$ ①

$= \$8139.27$ ①

(c) $\int_0^{20} f(t)\,dt = \frac{5}{2}(83+41+2(74+63+50))$ ①

$= 2\frac{1}{2} \times (124 + 187 \times 2)$

$= 1245$ ①

(d) (i) $r^2 + h^2 = 6^2$

$r^2 + h^2 = 36$ ①

(ii) $V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi(36-h^2) \times h$ from (i)

$V = 12\pi h - \frac{1}{3}\pi h^3$ ①

(iii) $\frac{dV}{dh} = 12\pi - \pi h^2 = 0$ for a maximum

$12 - h^2 = 0$

$h^2 = 12$

$h = \sqrt{12}$

$h = 2\sqrt{3}$ ①

$\frac{d^2V}{dh^2} = -2\pi h$

$= -2\pi \times 2\sqrt{3} < 0$

when $h = 2\sqrt{3}$ $\therefore$ ①

$h = 2\sqrt{3}$ gives a max. volume

Question 7. (a) $25^k (5^3)^4 = 1$

$(5^2)^k \times 5^{12} = 1$

$5^{2k+12} = 5^0$ ①

$\therefore 2k + 12 = 0$

$k = -6$ ①

(b) $A = \frac{1}{2}r^2(\theta - \sin\theta)$ ① area of segm[ent]

$= \frac{1}{2} \times 4^2(\frac{\pi}{3} - \sin\frac{\pi}{3})$ ① $60° = \frac{\pi}{3}$ c

$= 8(\frac{\pi}{3} - \frac{\sqrt{3}}{2})$ m^2 ①

or $8\left(\frac{2\pi - 3\sqrt{3}}{6}\right) = \frac{4(2\pi - 3\sqrt{3})}{3}$ m

(c) $\sum_{n=3}^{12}(2 \times 3^n) = 54 + 162$

$= 2(27 + 81 + \ldots\ldots)$ = $2 \times \frac{27(3^{10}-1)}{2}$ ①

18 terms ①

$27(3^{10}-$

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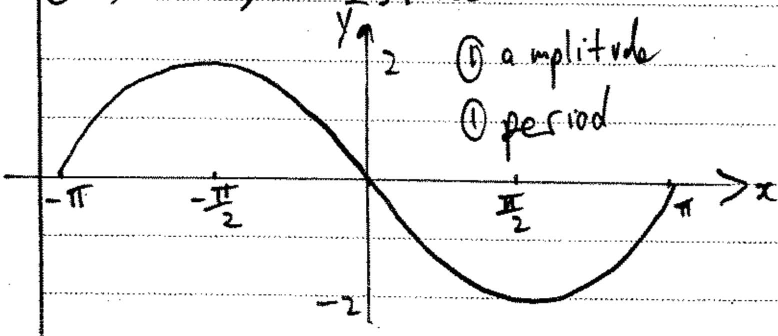
(d) (i) $f(x) = -2\sin x$



① amplitude
① period

(ii) Odd if

$-f(x) = f(-x)$

$-(-2\sin x) = -2\sin(-x)$

$2\sin x = -2 \times -\sin x$

$2\sin x = 2\sin x$ ✓ ①

$\therefore$ odd

(iii) $A = 2\int_{-\pi}^{0} -2\sin x \, dx$

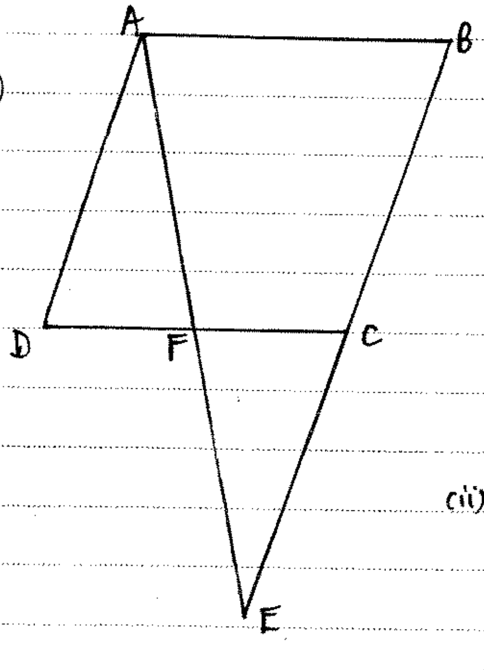
$= -4\left[-\cos x\right]_{-\pi}^{0}$ ①

$= 4\left[\cos 0 - \cos \pi\right]$

$= 4(1 - -1)$

$= 8 \text{ units}^2$ ①

Question 8: (a)



(i) In Δ's ADF and EBA,

∠D = ∠B (opposite angles of a rhombus) (

∠DAF = ∠FEC (alternate angles in parallel lines) (

$\therefore$ ΔADF ||| ΔEBA (equiangular)

(ii) Since EB : AD = 2:1 (EC = BC = A

$\therefore$ AB : DF = 2:1 (corresponding sides in similar triangle ①

But AB = DC (opposite sides of rhom

$\therefore$ DF : DC = 1 : 2 ①

so F must be the midpoint of DC.

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(b) (i) $M = M_0 e^{-kt}$

when $t=0$, $M = M_0$ $\therefore$ initial amt. is M_0

when $t=4$, $M = \frac{M_0}{4}$ ($\frac{3}{4}$ disintegrated)

$\therefore \frac{M_0}{4} = M_0 e^{-4k}$

$\frac{1}{4} = e^{-4k}$ (1)

$\log_e \frac{1}{4} = -4k$

$k = \frac{\log_e \frac{1}{4}}{-4}$

$\underline{k = 0.3465735}$ (1)

(ii) when $t = 90 = 1\frac{1}{2}$ hours, $M = 4$kg

$M = M_0 e^{-kt}$

$4 = M_0 e^{-0.3465735 \times 1\frac{1}{2}}$ (1)

$\underline{M_0 = 6.73\text{kg correct to 2 d.p.'s.}}$ (1)

(c) (i) $a = 6$

$v = 6t + c$

when $t=0$, $v=2$

$2 = 0 + c$

$\therefore c = 2$

$\therefore \underline{v = 6t + 2}$ (1)

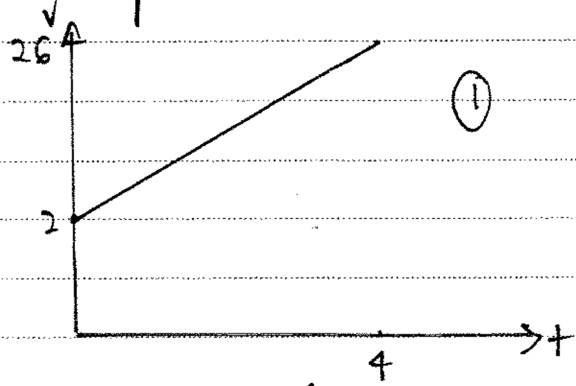
$x = 3t^2 + 2t + c$

when $t=0$, $x=2$

$\therefore c = 2$

$\underline{x = 3t^2 + 2t + 2}$ (1)

(ii)



(1)

(iii) distance $= \int_0^4 v \, dt$

= Area of trapezium

$= \frac{2+26}{2} \times 4$

$= 28 \times 2$

$\underline{= 56\text{m}}$ (1)

Question 9 (a) $A = \int$ top curve $-$ bottom curve dx.

Pts. of intersection at:

$x^2 + 6x = 8 - x^2$

$2x^2 + 6x - 8 = 0$

$x^2 + 3x - 4 = 0$

$(x-1)(x+4) = 0$ (1)

$\therefore A = \int_{-4}^{1} 8 - x^2 - (x^2 + 6x) \, dx$

$= \int_{-4}^{1} 8 - 2x^2 - 6x \, dx$

$= \left[8x - \frac{2}{3}x^3 - 3x^2 \right]_{-4}^{1}$ (1)

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$= 8 - \frac{2}{3} - 3 - \left(8 \times -4 - \frac{2}{3} \times -4^3 - 3 \times -4^2\right)$

$= 4\frac{1}{3} - \left(-32 + \frac{128}{3} - 48\right)$

$= \underline{41\frac{2}{3} \text{ units}^2}$ ①

ⓑ $T_n = ar^{n-1}$

$T_2 = 27 = ar^{2-1} \Rightarrow 27 = ar$ ①

$T_5 = 64 = ar^{5-1} \qquad 64 = ar^4$ ②

$\therefore$ ② ÷ ① gives $\frac{64}{27} = r^3$

$\therefore \underline{r = \frac{4}{3}}$ ① $\qquad \therefore 27 = a \times \frac{4}{3}$

$\underline{a = 20\frac{1}{4}}$ ①

c(ii) $S_5 = \frac{a(r^n - 1)}{r - 1}$

$= \frac{20\frac{1}{4}\left(\frac{4}{3}^5 - 1\right)}{\frac{4}{3} - 1}$ ①

$= \underline{195 \cdot 25}$ ①

ⓒ $y = \log_e\left(\frac{2x+1}{3x-1}\right)$

$= \log_e(2x+1) - \log_e(3x-1)$

$y' = \frac{2}{2x+1} - \frac{3}{3x-1}$ ①

$y' = \frac{2(3x-1) - 3(2x+1)}{(2x+1)(3x-1)}$

$= \frac{6x - 2 - 6x - 3}{(2x+1)(3x-1)}$

$= \underline{\frac{-5}{(2x+1)(3x-1)}}$ ①

ⓓ $\cos^2 2x = \frac{1}{4}$

$\cos 2x = \pm\frac{1}{2}$ ①

$2x = 60°, 120°, 240°, 300°, 420°, 48$[illegible] ① $600°, 6$[illegible]

$\underline{x = 30°, 60°, 120°, 150°, 210}$[illegible]

$\underline{240°, 300°, 330°}$ ①

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Question 10 (a) (i) $A = L^2$

$= (4t+20)^2$

$= 16t^2 + 160t + 400$ ①

(ii) $A = 784 = 16t^2 + 160t + 400$

$0 = 16t^2 + 160t - 384$

$0 = t^2 + 10t - 24$ ①

$0 = (t-2)(t+12)$ ①

$\therefore t = 2$ seconds as $t > 0$.

(iii) $\frac{dA}{dt} = 32t + 160$ ①

$\therefore$ when $t = 1$,

$\frac{dA}{dt} = 192\ cm^2/s$ ①

(b) (i) $A_1 = 100000 \times 1.005 - P$ ① as 6% p.a. $= 0.5\%$ p/month

$A_2 = A_1 \times 1.005 - P$

$= (100000 \times 1.005 - P) \times 1.005 - P$

$= 100000 \times 1.005^2 - P(1 + 1.005)$ as req'd ①

(ii) $A_n = 100000 \times 1.005^n - P(1 + 1.005 + 1.005^2 + \ldots\ldots 1.005^{n-1})$

G.P. $a = 1$, $r = 1.005$, $n = n$ ①

$= 100000 \times 1.005^n - \frac{P(1.005^n - 1)}{1.005 - 1}$ ①

$= 100000 \times 1.005^n - \frac{P(1.005^n - 1)}{0.005}$ ①

$= 100000 \times 1.005^n - 200P(1.005^n - 1)$ as req'd.

(iii) After 10 years $n = 120$

$\therefore A_{120} = 0 = 100000 \times 1.005^{120} - 200P(1.005^{120} - 1)$ ①

$\therefore P = \frac{100000 \times 1.005^{120}}{200(1.005^{120} - 1)}$

$P = \$1110.21$ a month ①

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